SOFIA: $\mathcal{MQ}$-based signatures in the QROM

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$\mathcal{MQ}$-based signatures

- Important candidate for post-quantum signatures
- Several submissions to NIST
  - DualModeMS [FPR17], GeMSS [CFMR+17], Gui [PCY+15, DCP+17a], HiMQ-3 [SPK17], LUOV [BPSV17], MQDSS [CHR+16, CHR+17], Rainbow [DS05, DCP+17b]
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- SOFIA: continue in line of MQDSS
  - Transform an $\mathcal{MQ}$-based IDS

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- No proof in the QROM
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- [KLP17]: tight Fiat-Shamir in the ROM
  - But similar issues in the QROM
- [KLS17]: Fiat-Shamir in QROM
  - Requires changing the IDS and parameters

This work

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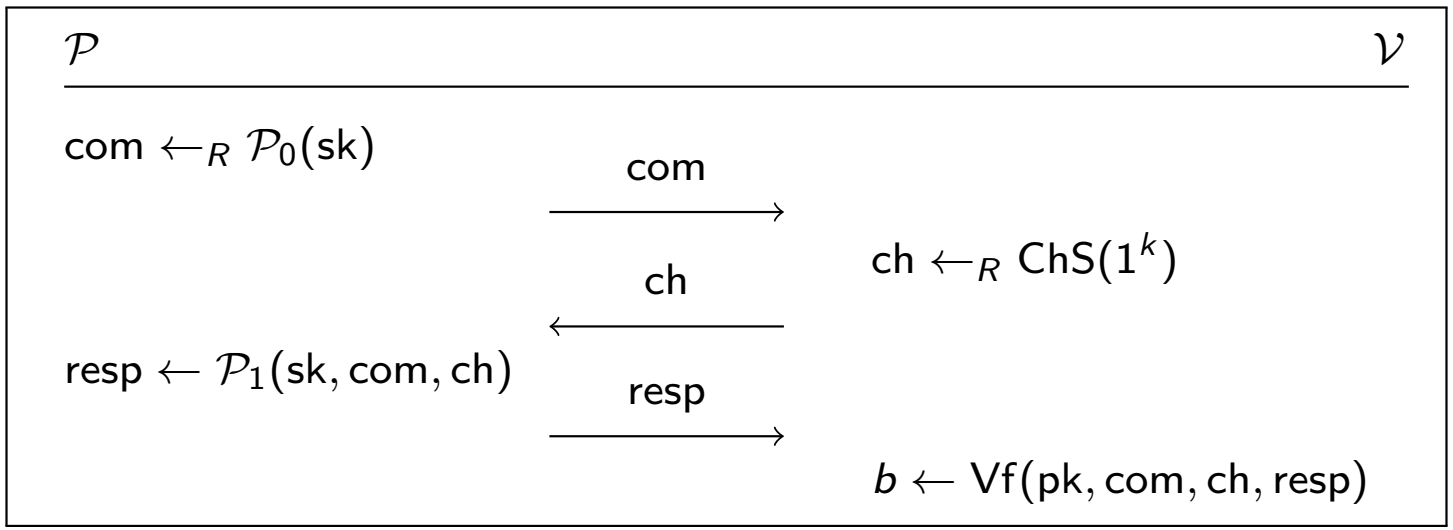
   - SOFIA-4-128
5. Implement and compare using Intel AVX2

Canonical Identification Schemes



Informally:

1. Prover commits to some (randomized) value derived from sk
2. Verifier picks a challenge 'ch'
3. Prover computes response 'resp'
4. Verifier checks if response matches challenge

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- Parallelize r rounds to decrease error
- Extra parameter: prepare for t challenges

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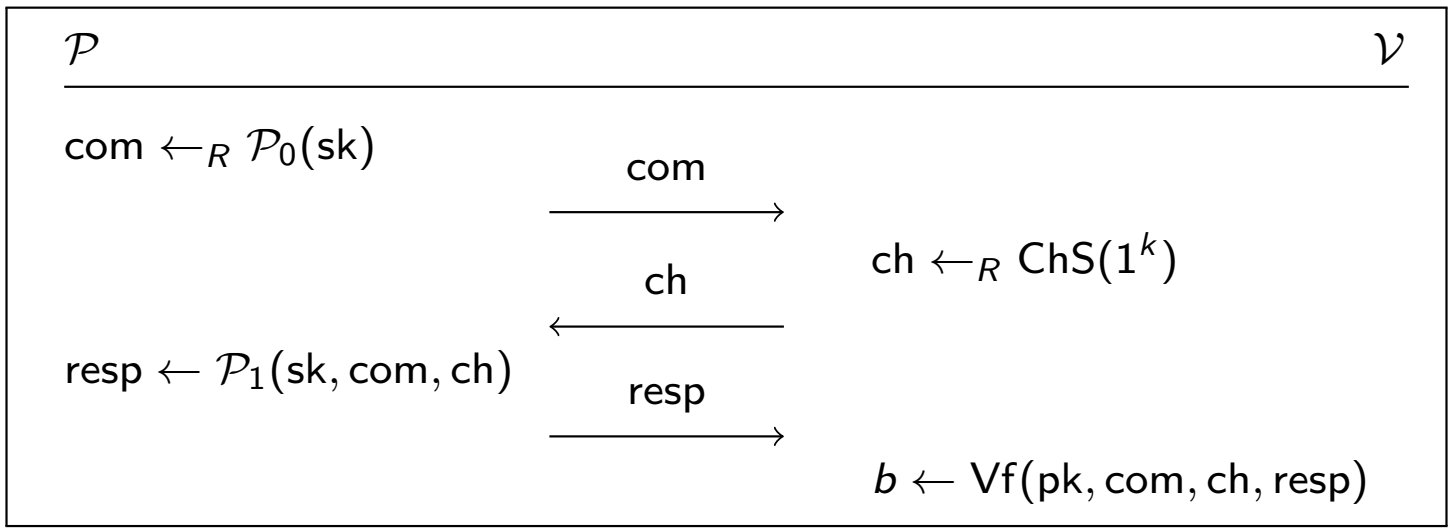
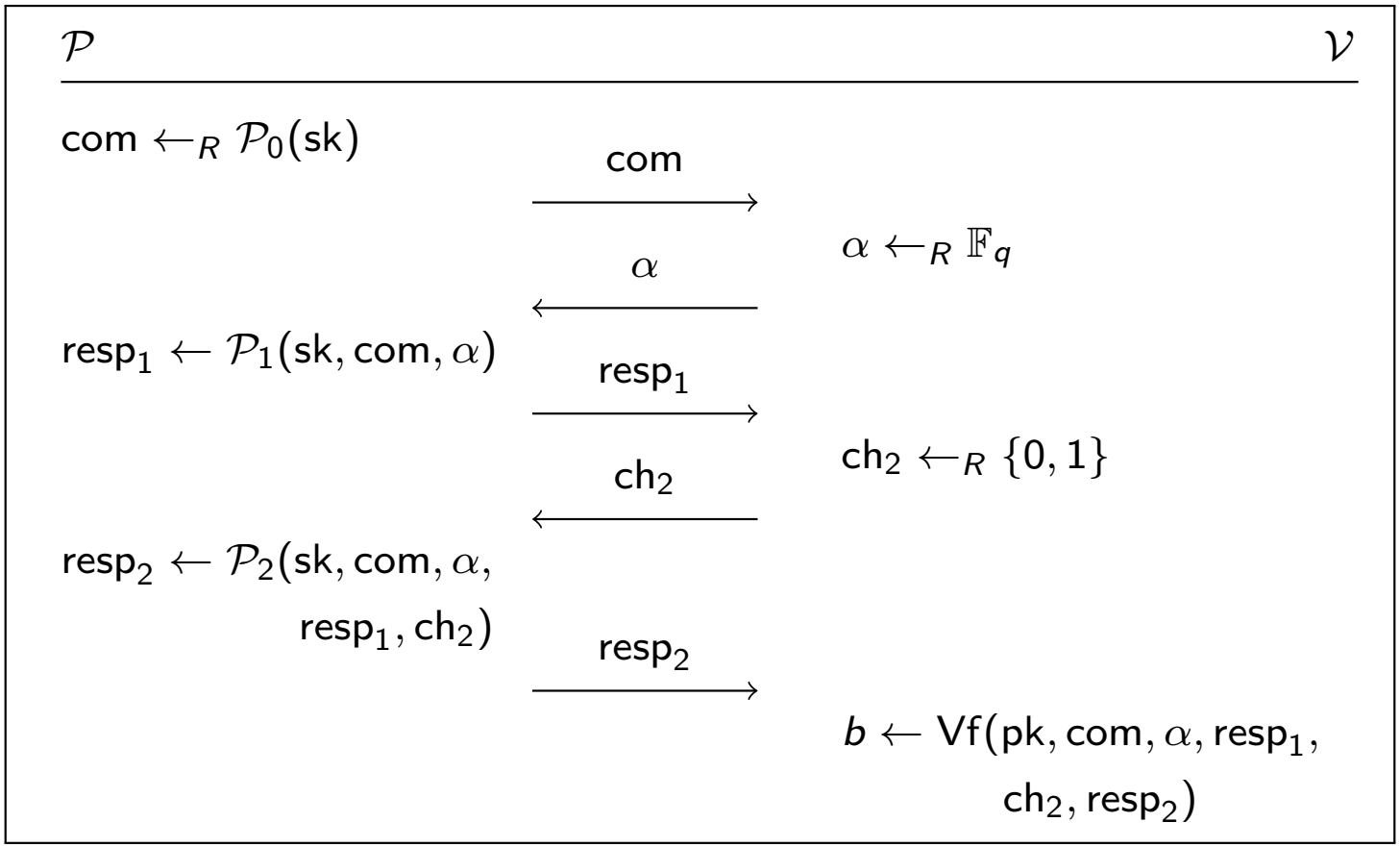


5-pass q2 Identification Schemes



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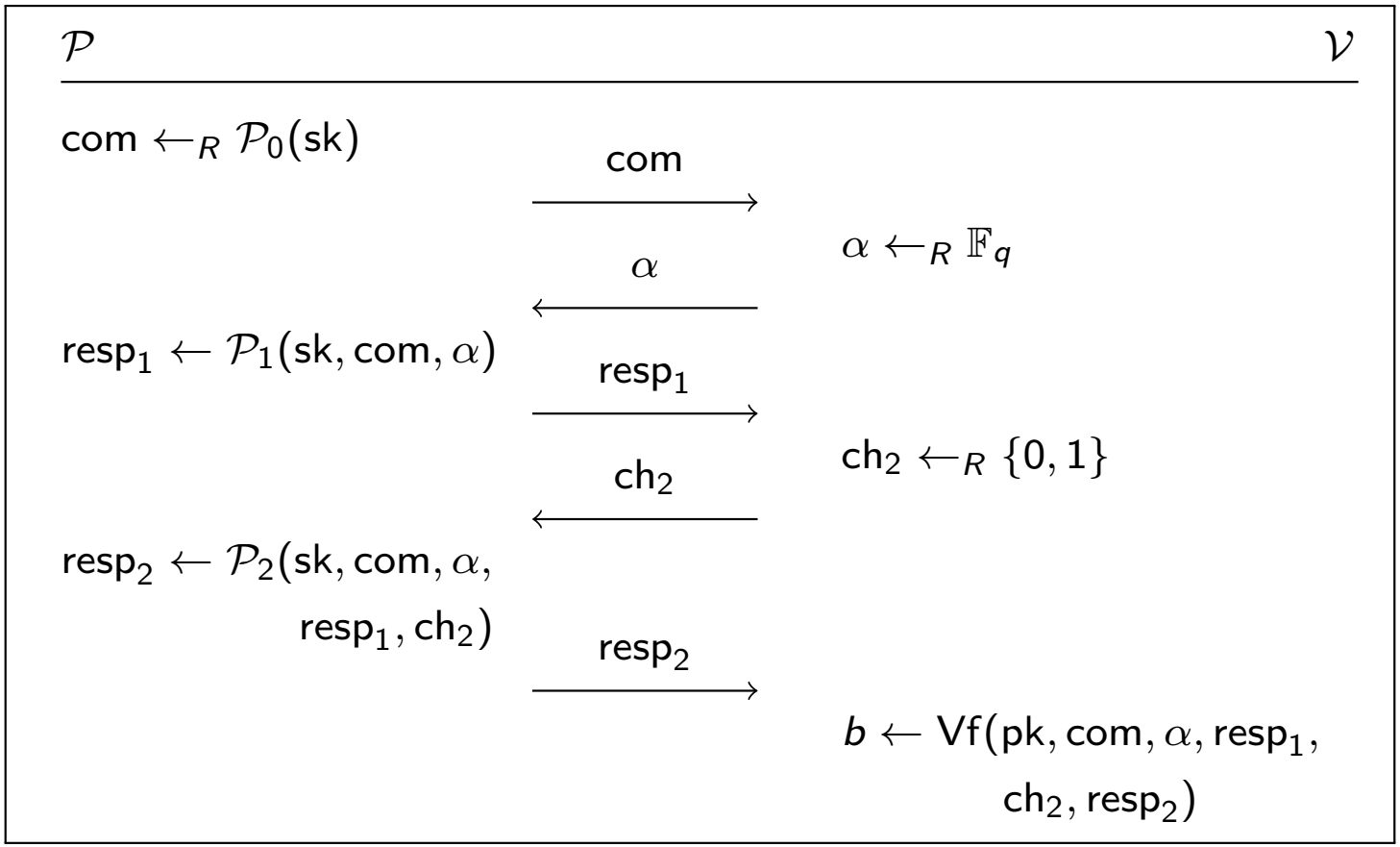

- Unruh's transform: resp_2 for both $\mathsf{ch}_2 \in \{0,1\}$, per α

$\mathcal{MQ}$ problem

The function family $\mathcal{MQ}(n, m, \mathbb{F}_q)$:

$$\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})), \text{ where } f_s(\mathbf{x}) = \sum_{i,j} a_{i,j}^{(s)} x_i x_j + \sum_i b_i^{(s)} x_i$$

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Problem: For given $\mathbf{y} \in \mathbb{F}_q^m$, find $\mathbf{x} \in \mathbb{F}_q^n$ such that $\mathbf{F}(\mathbf{x}) = \mathbf{y}$.

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i.e., solve the system of equations:

$$\begin{aligned} y_1 =& a_{1,1}^{(1)} x_1 x_1 + a_{1,2}^{(1)} x_1 x_2 + \ldots + a_{n,n}^{(1)} x_n x_n + b_1^{(1)} x_1 + \ldots + b_n^{(1)} x_n \\ & \vdots \\ y_m =& a_{1,1}^{(m)} x_1 x_1 + a_{1,2}^{(m)} x_1 x_2 + \ldots + a_{n,n}^{(m)} x_n x_n + b_1^{(m)} x_1 + \ldots + b_n^{(m)} x_n \end{aligned}$$

Sakumoto-Shirai-Hiwatari 5-pass IDS [SSH11]

$\mathcal{P} : (\mathbf{F}, \mathbf{v}, \mathbf{s})$		$\mathcal{V} : (\mathbf{F}, \mathbf{v})$
$\mathbf{r}_0, \mathbf{t}_0 \leftarrow_R \mathbb{F}_q^n, \mathbf{e}_0 \leftarrow_R \mathbb{F}_q^m$		
$\mathbf{r}_1 \leftarrow \mathbf{s} - \mathbf{r}_0$		
$c_0 \leftarrow \mathit{Com}(\mathbf{r}_0, \mathbf{t}_0, \mathbf{e}_0)$		
$c_1 \leftarrow \mathit{Com}(\mathbf{r}_1, \mathbf{G}(\mathbf{t}_0, \mathbf{r}_1) + \mathbf{e}_0)$	$\xrightarrow{(c_0, c_1)}$	
	$\xleftarrow{\alpha}$	$\alpha \leftarrow_R \mathbb{F}_q$
$\mathbf{t}_1 \leftarrow \alpha\mathbf{r}_0 - \mathbf{t}_0$		
$\mathbf{e}_1 \leftarrow \alpha\mathbf{F}(\mathbf{r}_0) - \mathbf{e}_0$	$\xrightarrow{\mathsf{resp}_1 = (\mathbf{t}_1, \mathbf{e}_1)}$	
	$\xleftarrow{\mathsf{ch}_2}$	$\mathsf{ch}_2 \leftarrow_R \{0, 1\}$
If $\mathsf{ch}_2 = 0$, $\mathsf{resp}_2 \leftarrow \mathbf{r}_0$		
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(evaluating $\mathbf{G} \approx$ evaluating $\mathbf{F}$)

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Key generation:

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Signing:

- Run the transformed IDS r times in parallel
  - Commit to randomness; $r \times \mathbf{G}$
  - Respond to t challenges $\alpha \in \mathbb{F}_q$; $r \times t \times \mathbf{F}$
- Hash (blinded) responses to set of indices
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Verification:

- Reconstruct indices, responses, commitments
- Verify revealed responses
- Verify that commitments match responses; $r \times \mathbf{F}$, $\backsim \frac{1}{2} r \times \mathbf{G}$

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- $t = 3$, $r = 438$ (since $2^{-(r \log \frac{2t}{t+1})/2} < 2^{-128}$)

- XOFs, hashes, PRGs: SHAKE, cSHAKE, (AES)

Implementation

- Evaluating $\mathcal{MQ}$

- XOFs

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- Evaluating $\mathcal{MQ}$
  - 438 rounds, 2x per round
  - Pairwise multiply 128x $\in \mathbb{F}_4$
  - Multiply by coefficients from $\mathbf{F}$, $\in \mathbb{F}_4$
  - Accumulate

- XOFs
  - Blinding commitments
  - Expanding $\mathbf{F}$: 262 KiB

  - External parallelism and cSHAKE

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- From $\mathbf{x}$ to $\mathbf{F}(\mathbf{x})$ should be easy

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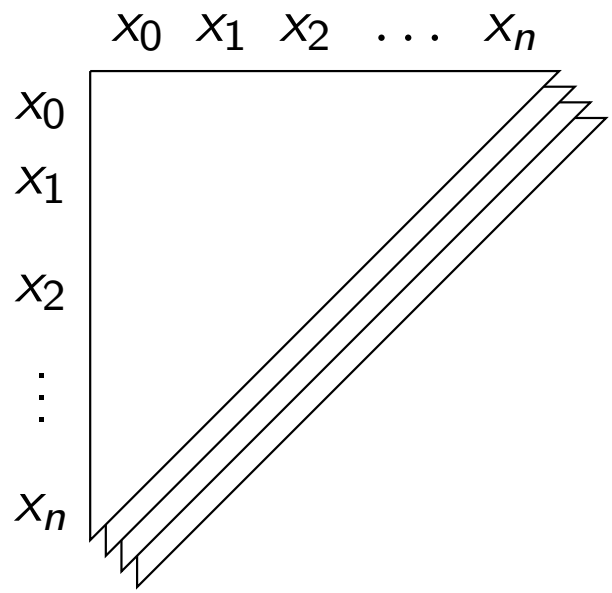

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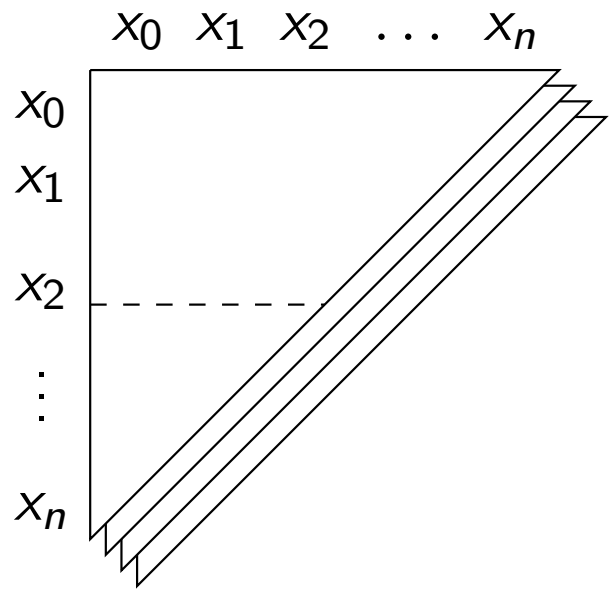

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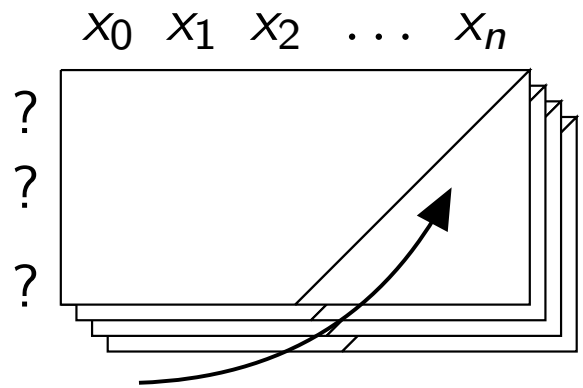

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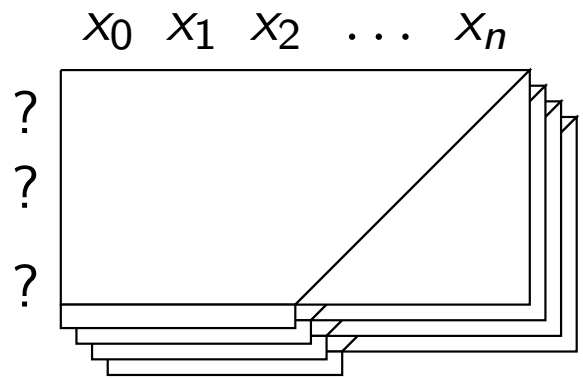

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  $\Rightarrow$ very fast multiplication

$$c_{high} = (a_{high} \wedge (b_{high} \oplus b_{low})) \oplus (a_{low} \wedge b_{high})$$
$$c_{low} = (a_{low} \wedge b_{low}) \oplus (a_{high} \wedge b_{high})$$

- `vpand`, `vpand`, `vpermq`, `vpxor`

SOFIA-4-128 vs MQDSS-31-64

a.k.a. the price of QROM

- Signature size: 123 KiB (MQDSS: 40 KiB)
- 64 bytes pk, 32 bytes sk (MQDSS: 72 B, 64 B)

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- 64 bytes pk, 32 bytes sk (MQDSS: 72 B, 64 B)

- Key generation 1.16 M cycles (MQDSS: 1.18 M)
- Signing 21.31 M cycles (MQDSS: 8.51 M)
  - $\backsim$75% $\mathcal{MQ}$
  - $\backsim$25% SHAKE
- Verification 15.49 M cycles (MQDSS: 5.75 M)

(Intel Haswell, Core-i7-4770K, AVX2)

Conclusions and comparisons

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- C and AVX2 code available (public domain):
  https://joostrijneveld.nl/papers/sofia

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- Key technique: cut-and-choose for $\mathcal{MQ}$
  - Analogously, consider DLP: $s = r_0 + r_1 \Rightarrow g^s = g^{r_0} \cdot g^{r_1}$

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  - Split $\mathbf{s}$ and $\mathbf{F}(\mathbf{s})$ into $\mathbf{r}_0, \mathbf{r}_1$ and $\mathbf{F}(\mathbf{r}_0), \mathbf{F}(\mathbf{r}_1)$
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  - Split $\mathbf{r}_0$ and $\mathbf{F}(\mathbf{r}_0)$ further into $\mathbf{t}_0, \mathbf{t}_1$ resp. $\mathbf{e}_0, \mathbf{e}_1$

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  - Takeaway: evaluating $\mathbf{G} \approx$ evaluating $\mathbf{F}$
- Result: reveal either $\mathbf{r}_0$ or $\mathbf{r}_1$, and $(\mathbf{t}_0, \mathbf{e}_0)$ or $(\mathbf{t}_1, \mathbf{e}_1)$

Optimizations

Many similarities to e.g. Picnic [CDG+17]

- Exclude redundant blinded responses
- Fix challenge space to $|\mathsf{ChS}_1| = t$
- Unlink α and ch_2
- Omit commitments [SSH11]
- Self-randomizing commitments

What doesn't help:

- Opening for multiple α
- Committing to multiple $\mathbf{t}_0$

Evaluating $\mathcal{MQ}$, cont.

- 'Vertically:' broadcast monomial, multiply with **F**
  - $a^{(1)}_{1,1} x_1 x_1, a^{(\mathbf{2})}_{1,1} x_1 x_1, a^{(\mathbf{3})}_{1,1} x_1 x_1, a^{(4)}_{1,1} x_1 x_1, \ldots$
- 'Horizontally:' iterate over output elements, `popcnt`
  - $a^{(1)}_{1,1} x_1 x_1, a^{(1)}_{1,\mathbf{2}} x_1 x_{\mathbf{2}}, a^{(1)}_{1,\mathbf{3}} x_1 x_{\mathbf{3}}, \ldots a^{(1)}_{2,1} x_2 x_1, a^{(1)}_{2,2} x_2 x_2, \ldots$

Evaluating $\mathcal{MQ}$, cont.

- 'Vertically:' broadcast monomial, multiply with **F**
  - $a^{(1)}_{1,1}x_1x_1, a^{(\mathbf{2})}_{1,1}x_1x_1, a^{(\mathbf{3})}_{1,1}x_1x_1, a^{(4)}_{1,1}x_1x_1, \ldots$
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- Horizontal: more loads, but internal parallelism

Evaluating $\mathcal{MQ}$, cont.

- 'Vertically:' broadcast monomial, multiply with $\mathbf{F}$
  - $a^{(1)}_{1,1}x_1x_1, a^{(\mathbf{2})}_{1,1}x_1x_1, a^{(\mathbf{3})}_{1,1}x_1x_1, a^{(4)}_{1,1}x_1x_1, \ldots$
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- Horizontal: more loads, but internal parallelism
- Both cases: delay reductions in $\mathbb{F}_4$
  - $[\hat{\mathbf{x}}_{high} \wedge \mathbf{f}_{high} | \hat{\mathbf{x}}_{low} \wedge \mathbf{f}_{low}]$ and $[\hat{\mathbf{x}}_{low} \wedge \mathbf{f}_{high} | \hat{\mathbf{x}}_{high} \wedge \mathbf{f}_{low}]$

Evaluating $\mathcal{MQ}$, cont.

- 'Vertically:' broadcast monomial, multiply with $\mathbf{F}$
  - $a^{(1)}_{1,1}x_1x_1, a^{(\mathbf{2})}_{1,1}x_1x_1, a^{(\mathbf{3})}_{1,1}x_1x_1, a^{(4)}_{1,1}x_1x_1, \ldots$
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- Both cases: delay reductions in $\mathbb{F}_4$
  - $[\hat{\mathbf{x}}_{high} \wedge \mathbf{f}_{high} | \hat{\mathbf{x}}_{low} \wedge \mathbf{f}_{low}]$ and $[\hat{\mathbf{x}}_{low} \wedge \mathbf{f}_{high} | \hat{\mathbf{x}}_{high} \wedge \mathbf{f}_{low}]$
- Both cases: external parallelism over constant $\mathbf{F}$
- Horizontal in batches of 3, avg. 17 558 cycles per $\mathcal{MQ}$